

NAG Toolbox for MATLAB

f07ha

1 Purpose

f07ha computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric positive-definite band matrix of bandwidth $(2k_d + 1)$ and X and B are n by r matrices.

2 Syntax

```
[ab, b, info] = f07ha(uplo, kd, ab, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07ha uses the Cholesky decomposition to factor A as $A = U^T U$ if **uplo** = 'U' or $A = LL^T$ if **uplo** = 'L', where U is an upper triangular band matrix, and L is a lower triangular band matrix, with the same number of superdiagonals or subdiagonals as A . The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

If **uplo** = 'U', the upper triangle of A is stored.

If **uplo** = 'L', the lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **kd** – int32 scalar

k_d , the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0 .

3: **ab(ldab,*)** – double array

The first dimension of the array **ab** must be at least **kd** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of the symmetric band matrix A .

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**($k_d + 1 + i - j, j$) for $\max(1, j - k_d) \leq i \leq j$;
 if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**($1 + i - j, j$) for $j \leq i \leq \min(n, j + k_d)$.

4: **b(ldb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, nrhs_p)$

The n by r right-hand side matrix B .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **ab**.

n , the number of linear equations, i.e., the order of the matrix A .

Constraint: $n \geq 0$.

2: **nrhs_p** – int32 scalar

Default: The second dimension of the array **b**.

r , the number of right-hand sides, i.e., the number of columns of the matrix B .

Constraint: **nrhs_p** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldb

5.4 Output Parameters

1: **ab(ldab,*)** – double array

The first dimension of the array **ab** must be at least $k_d + 1$

The second dimension of the array must be at least $\max(1, n)$

If **info** = 0, the triangular factor U or L from the Cholesky factorization $A = U^T U$ or $A = LL^T$ of the band matrix A , in the same storage format as A .

2: **b(ldb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, nrhs_p)$

If **info** = 0, the n by r solution matrix X .

3: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kd**, 4: **nrhs_p**, 5: **ab**, 6: **ldab**, 7: **b**, 8: **ldb**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , the leading minor of order i of A is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

f07hb is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, f04bf solves $Ax = b$ and returns a forward error bound and condition estimate. f04bf calls f07ha to solve the equations.

8 Further Comments

When $n \gg k$, the total number of floating-point operations is approximately $n(k+1)^2 + 4nkr$, where k is the number of superdiagonals and r is the number of right-hand sides.

The complex analogue of this function is f07hn.

9 Example

```
uplo = 'U';
kd = int32(1);
ab = [0, 2.68, -2.39, -2.22;
      5.49, 5.63, 2.6, 5.17];
b = [22.09;
     9.31;
     -5.24;
     11.83];
[abOut, bOut, info] = f07ha(uplo, kd, ab, b)

abOut =
      0      1.1438     -1.1497     -1.9635
    2.3431     2.0789      1.1306      1.1465
bOut =
    5.0000
   -2.0000
   -3.0000
    1.0000
info =
      0
```

